Computação Gráfica

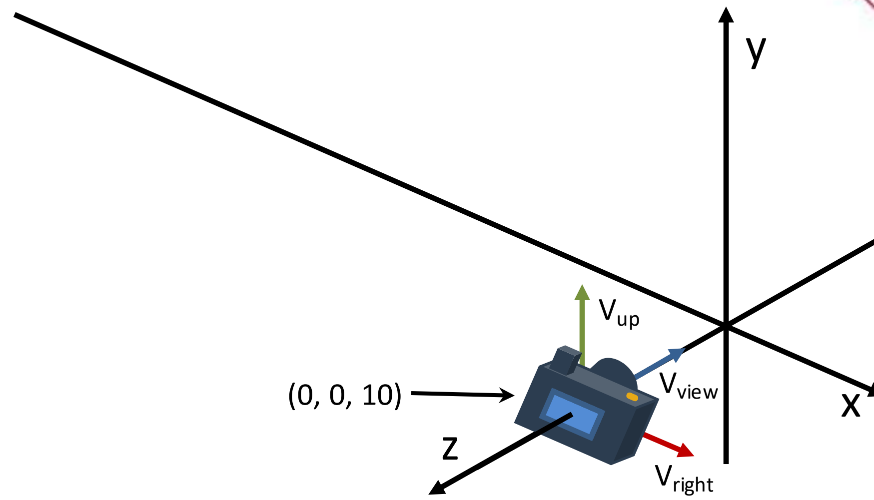
Aula 7: Revisão

Exemplo Completo:

- Transformações Geométricas
- Quatérnios
- Transformação Look-at
- Transformação Perspectiva
- Coordenadas normalizadas (NDC)
- Divisão Homogênea
- Transformação de tela
- Supersampling

Definindo posição da câmera virtual

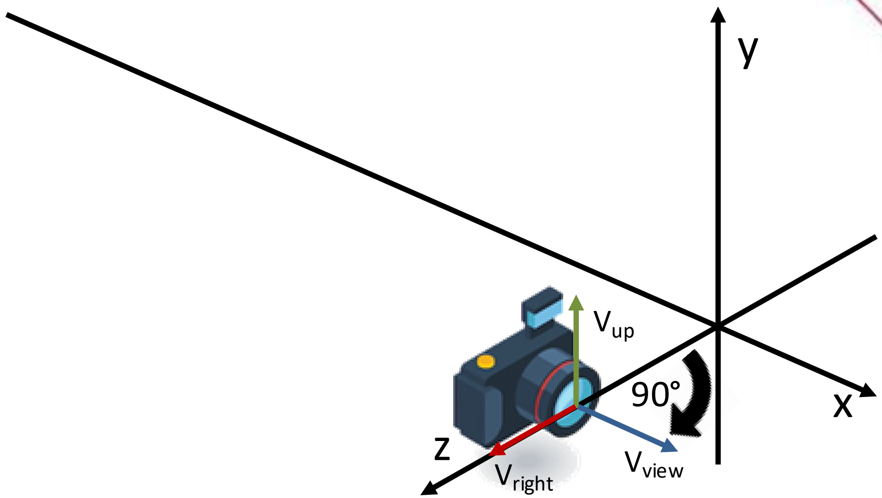
<Scene>
<Viewpoint/>
</Scene>



https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/components/navigation.html

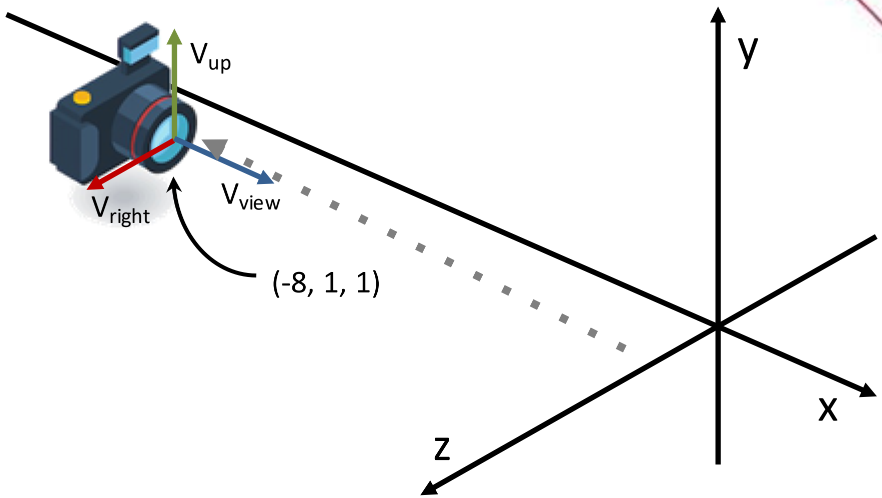
Definindo posição da câmera virtual

```
<Scene>
  <Viewpoint orientation="0 1 0 -1.57"/>
</Scene>
```



https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/components/navigation.html

Definindo posição da câmera virtual

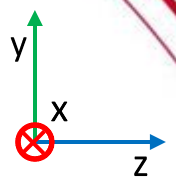



```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
</Scene>
```

https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/components/navigation.html

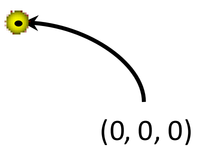
Criando uma esfera no centro do mundo

Visualização do ponto de vista da câmera mas no sistema de coordenadas do mundo.




```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

amarelo



centro da esfera : $[0 \quad 0 \quad 0 \quad 1]^T$

ou

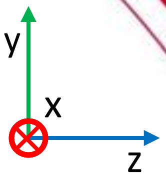
centro da esfera : $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

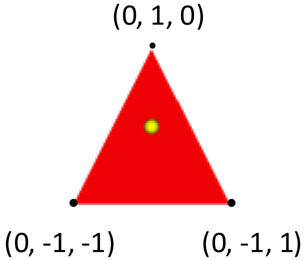
https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/components/group.html

Criando um triângulo no centro do mundo

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

vermelho





P_0 P_1 P_2

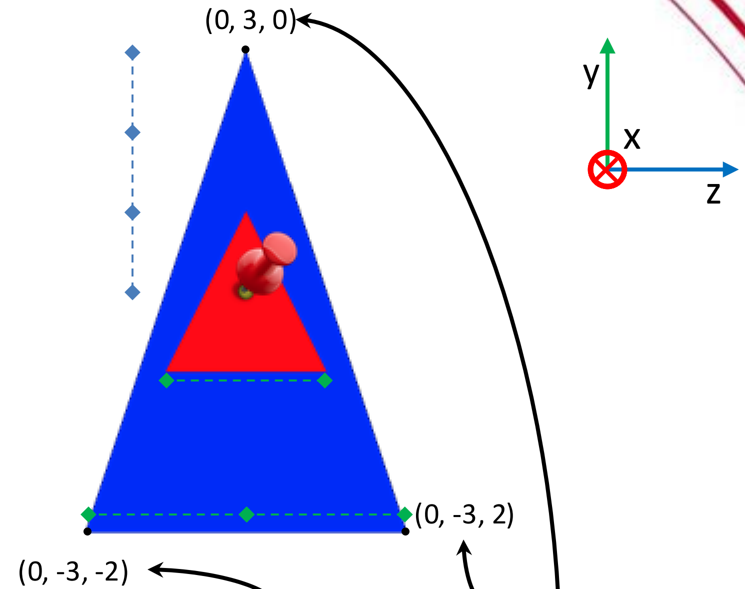
$$\text{vértices do triângulo} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/components/rendering.html

Novo triângulo ampliado

```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

azul



$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 3 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 1\end{bmatrix} \times \begin{bmatrix}0 & 0 & 0\\-1 & -1 & 1\\-1 & 1 & 0\\1 & 1 & 1\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\-3 & -3 & 3\\-2 & 2 & 0\\1 & 1 & 1\end{bmatrix}$$

https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/components/rendering.html

Novo triângulo rotacionado

```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

$$\mathbf{R}_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)u_x i + \sin\left(\frac{\theta}{2}\right)u_y j + \sin\left(\frac{\theta}{2}\right)u_z k$$

$$q = \cos(-0.79) + \sin(-0.79)1i + \sin(-0.79)0j + \sin(-0.79)0k$$

$$q = 0.71 - 0.71i$$

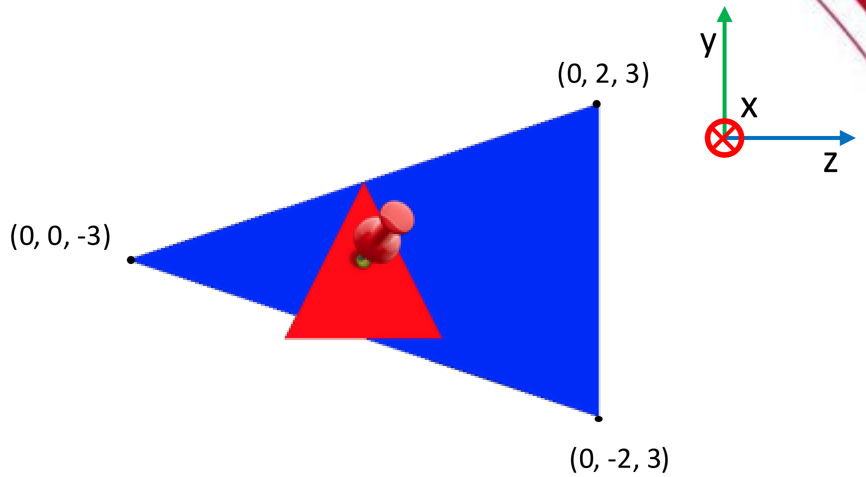
$$R = \begin{bmatrix} 1-2\left(q_j^2+q_k^2\right) & 2(q_iq_j - q_kq_r) & 2(q_iq_k + q_jq_r) & 0 \\ 2(q_iq_j + q_kq_r) & 1-2\left(q_i^2+q_k^2\right) & 2(q_jq_k - q_iq_r) & 0 \\ 2(q_iq_k - q_jq_r) & 2(q_jq_k + q_iq_r) & 1-2\left(q_i^2+q_j^2\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/fieldsDef.html#SFRotationAndMFRotation

Novo triângulo rotacionado

```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



rotação escala

$$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}1&0&0&0\\0&3&0&0\\0&0&2&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}\mathbf{1}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{2}&\mathbf{0}\\\mathbf{0}&\mathbf{-3}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{1}\end{bmatrix}$$

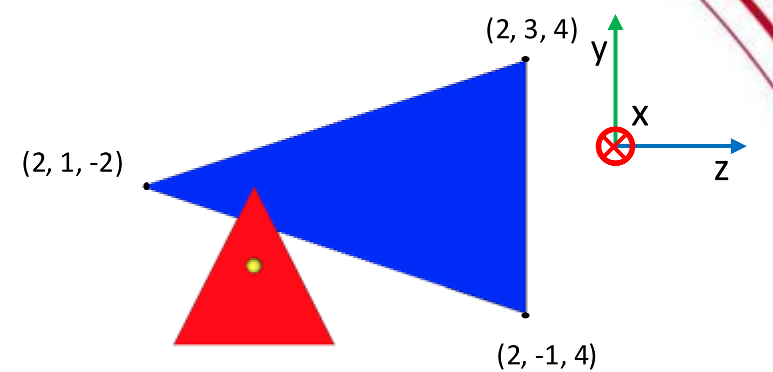
$$\begin{bmatrix}1&0&0&0\\0&0&2&0\\0&-3&0&0\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}0&0&0\\-1&-1&1\\-1&1&0\\1&1&1\end{bmatrix}=\begin{bmatrix}0&0&0\\-2&2&0\\3&3&-3\\1&1&1\end{bmatrix}$$

https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/fieldsDef.html#SFRotationAndMFRotation

Insper

Novo triângulo transladando

```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



translação · rotação · escala

$$\begin{bmatrix}1&0&0&2\\0&1&0&1\\0&0&1&1\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}1&0&0&0\\0&3&0&0\\0&0&2&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}\mathbf{1}&\mathbf{0}&\mathbf{0}&\mathbf{2}\\\mathbf{0}&\mathbf{0}&\mathbf{2}&\mathbf{1}\\\mathbf{0}&\mathbf{-3}&\mathbf{0}&\mathbf{1}\\\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{1}\end{bmatrix}$$

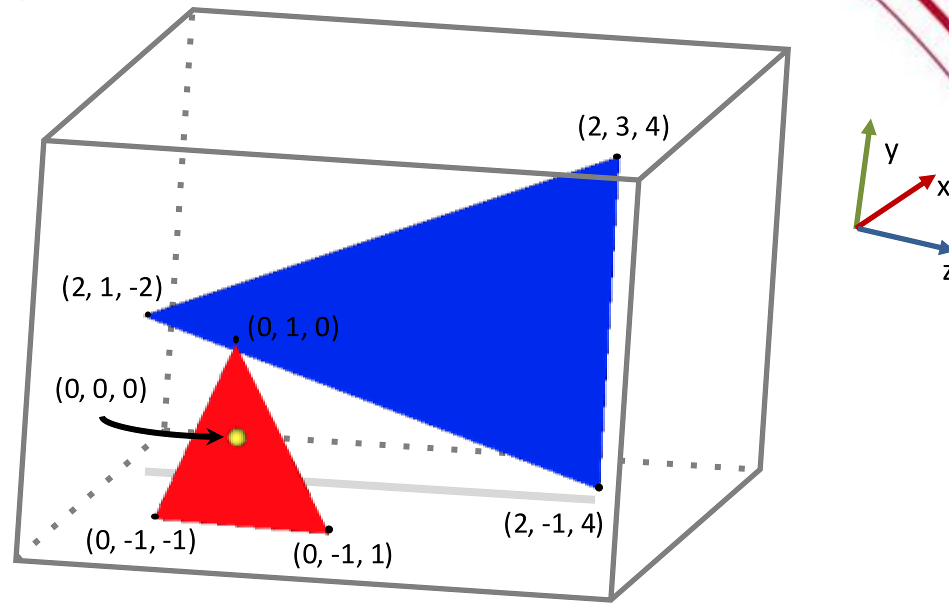
$$\begin{bmatrix}1&0&0&2\\0&0&2&1\\0&-3&0&1\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}0&0&0\\-1&-1&1\\-1&1&0\\1&1&1\end{bmatrix}=\begin{bmatrix}2&2&2\\-1&3&1\\4&4&-2\\1&1&1\end{bmatrix}$$

https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/fieldsDef.html#SFVec3fAndMFVec3f

Insper

Visualizando de outro ângulo

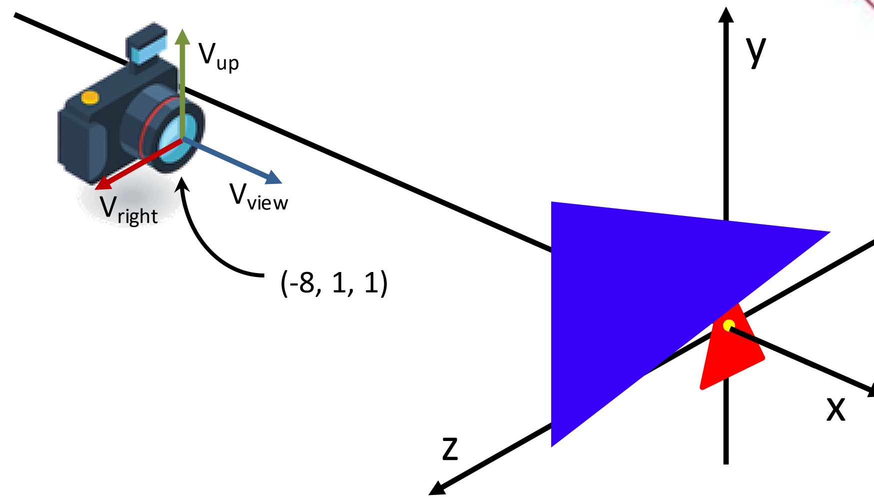
```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



$$\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \quad \begin{bmatrix}0 & 0 & 0\\-1 & -1 & 1\\-1 & 1 & 0\\1 & 1 & 1\end{bmatrix} \quad \begin{bmatrix}2 & 2 & 2\\-1 & 3 & 1\\4 & 4 & -2\\1 & 1 & 1\end{bmatrix}$$

Transformação da Câmera

```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



u = V_{right}
v = V_{up}
w = V_{view}

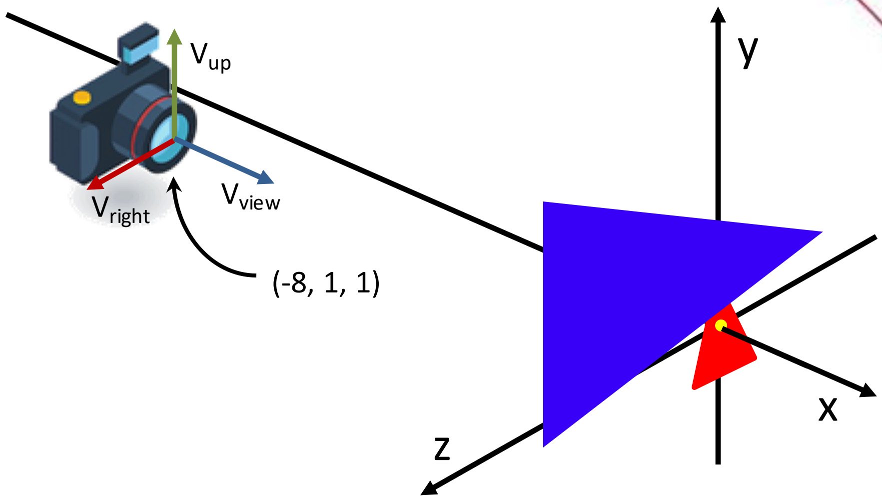
$$\begin{bmatrix} u_x & v_x & -w_x & e_x \\ u_y & v_y & -w_y & e_y \\ u_z & v_z & -w_z & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$(T \cdot R)^{-1} = R^{-1} \cdot T^{-1}$$

Insper

Transformação da Câmera (Rotação)

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



Rotação:

Como temos uma câmera que estava na base da cena (ou seja identidade), podemos usar diretamente a rotação realizada para calcular a nova matriz.

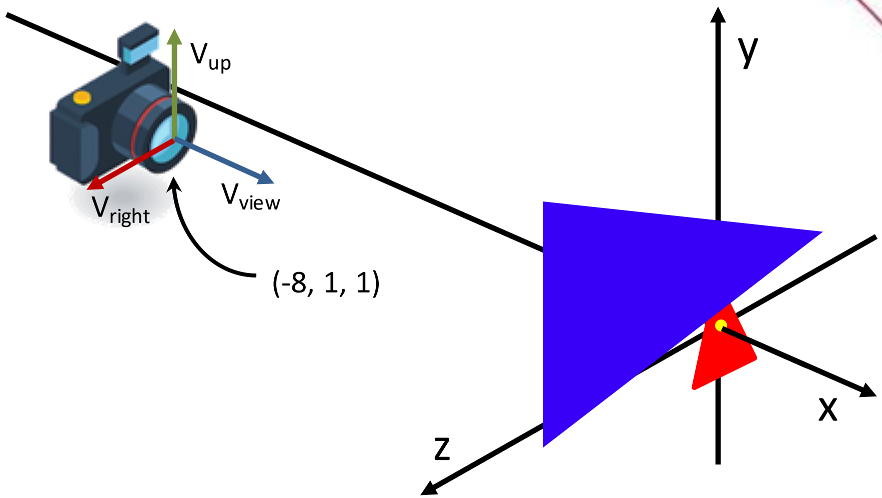
$$q = \cos(-0.79) + \sin(-0.79)0i + \sin(-0.79)1j + \sin(-0.79)0k$$
$$q = 0.71 - 0.71j$$

$$R = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Insper

Transformação da Câmera (Translação)

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

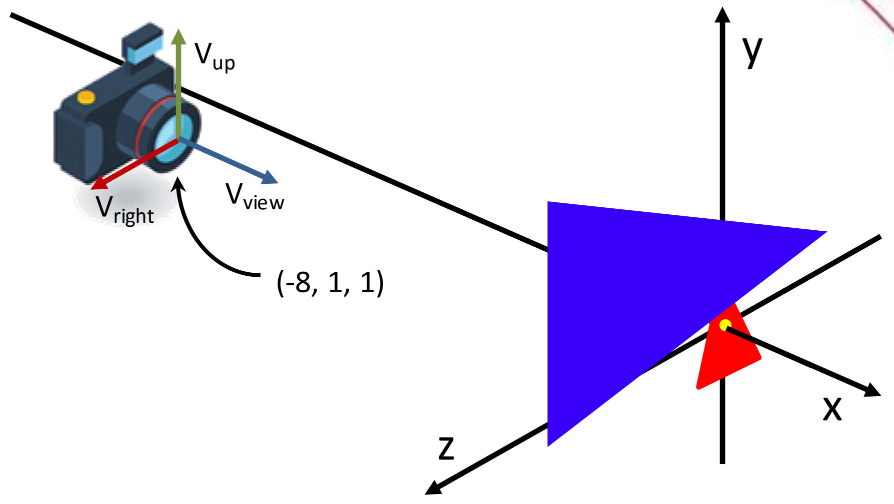


Translação:

$$T = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformação da Câmera

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```

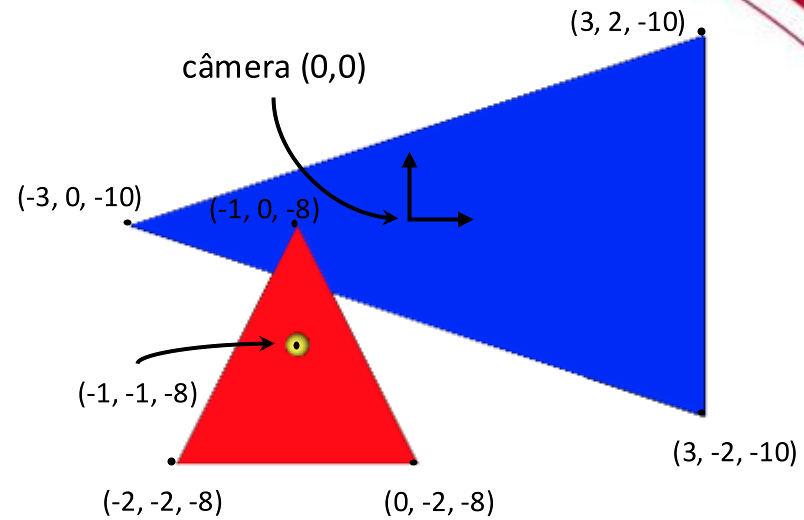


Orientação^{-1} × Translação^{-1} = **Matriz de Visualização (view)**

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{-8} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Aplicando Transformação

```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



Matriz de Visualização

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -8 \\ 1 \end{bmatrix}$$

Matriz de Visualização

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ -2 & -2 & 0 \\ -8 & -8 & -8 \\ 1 & 1 & 1 \end{bmatrix}$$

Matriz de Visualização

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 2 \\ -1 & 3 & 1 \\ 4 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -3 \\ -2 & 2 & 0 \\ -10 & -10 & -10 \\ 1 & 1 & 1 \end{bmatrix}$$

Matriz Perspectiva

```xml
<Scene>
 <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
 <Transform>
  <Shape>
   <Sphere radius='0.1'/>
   <Appearance>
    <Material diffuseColor='1 1 0'/>
   </Appearance>
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 </Transform>
 <Transform>
  <Shape>
   <TriangleSet>
    <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
   </TriangleSet>
   <Appearance>
    <Material diffuseColor='1 0 0'/>
   </Appearance>
  </Shape>
 </Transform>
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        rotation="1 0 0 -1.57"
        translation="2 1 1">
  <Shape>
   <TriangleSet>
    <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
   </TriangleSet>
   <Appearance>
    <Material diffuseColor='0 0 1'/>
   </Appearance>
  </Shape>
 </Transform>
</Scene>
```

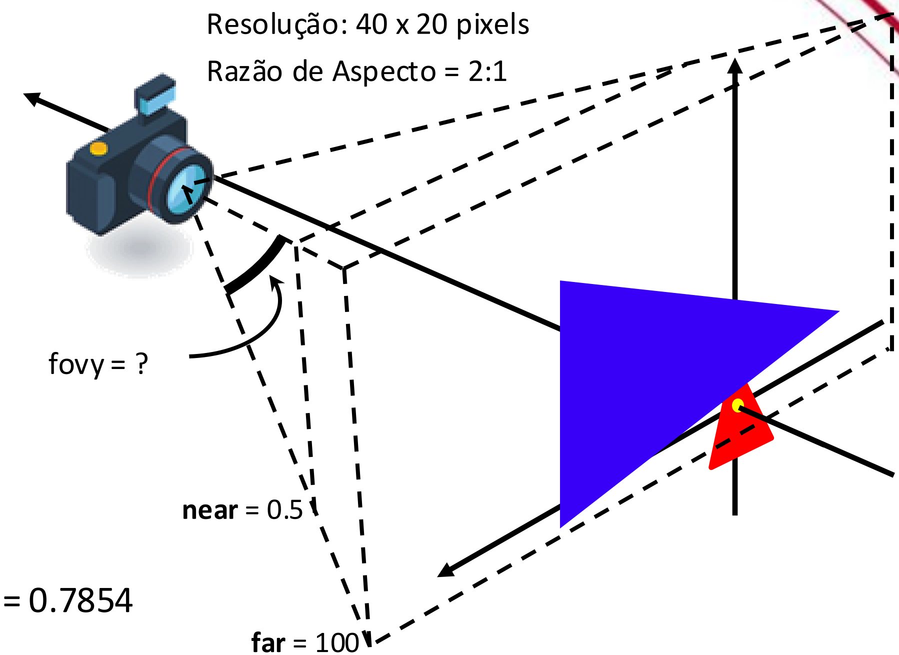

fielOfView = $\pi/4$ = 0.7854

$$\mathrm{FOV}y = 2 \cdot \arctan\left(\tan\left(\frac{FOVd}{2}\right) \cdot \frac{\mathrm{Altura}}{\sqrt{\mathrm{Altura}^2 + \mathrm{Largura}^2}}\right)$$

$$\mathrm{FOV}y = 2 \cdot \arctan\left(\tan\left(\frac{0.7854}{2}\right) \cdot \frac{20}{\sqrt{20^2 + 40^2}}\right)$$

$$\mathrm{FOV}y = 0.3663$$

https://www.web3d.org/documents/specifications/19775-1/V3.3/Part01/components/navigation.html

Matriz Perspectiva

```
<Scene>
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      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```
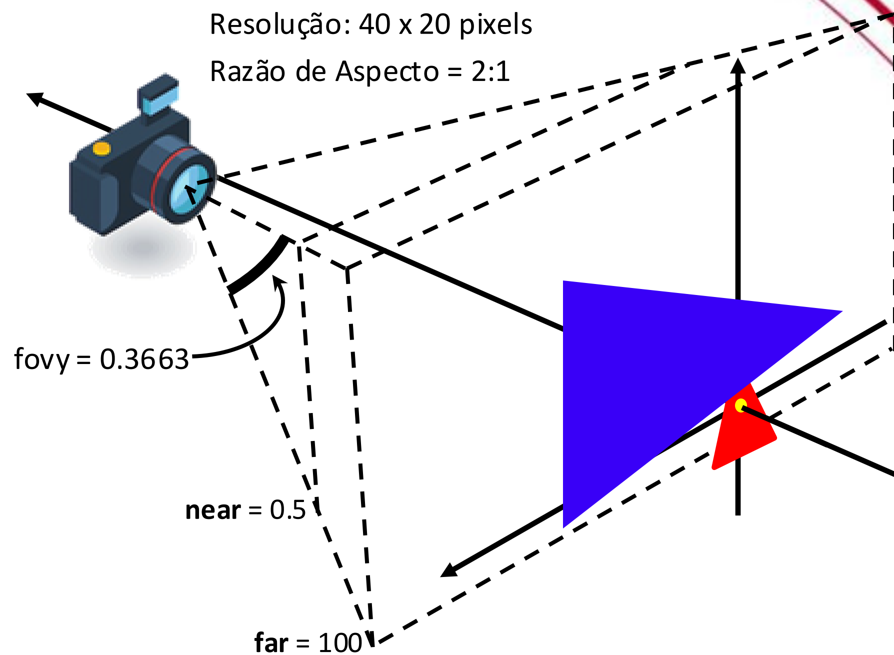



$$\mathbf{P} = \begin{bmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & \frac{-2far*near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

top = near * tan (fovy) = 0.5 * 0.3663 = 0.1832

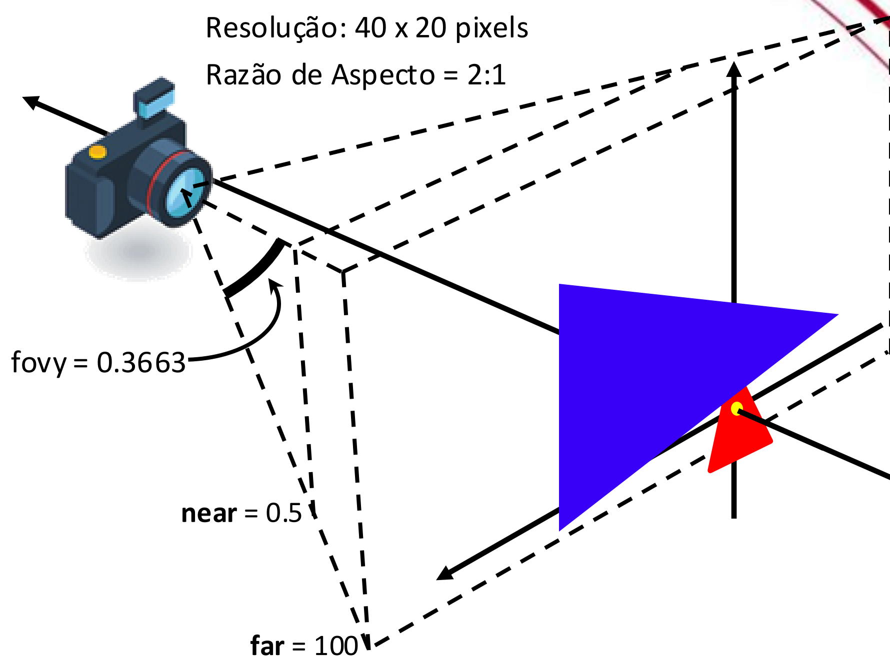
bottom = –top = –top = -0.1832

right = top * aspect = top * aspect = 0.3663

left = –right = –right = -0.3663

Matriz Perspectiva

```
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



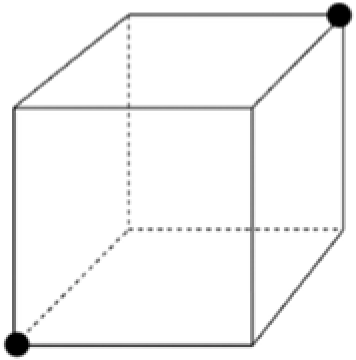
$$\mathbf{P} = \begin{bmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & \frac{-2far*near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

top = 0.1832
bottom = -0.1832
right = 0.3663
left = -0.3663

$$P = \begin{bmatrix} \frac{0.5}{0.3663} & 0 & 0 & 0 \\ 0 & \frac{0.5}{0.1832} & 0 & 0 \\ 0 & 0 & -\frac{100+0.5}{100-0.5} & \frac{-2*100*0.5}{100-0.5} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{1.3649} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{2.7298} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1.01} & \mathbf{-1.005} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{0} \end{bmatrix}$$

Aplicando Matriz de Projeção Perspectiva

Projeção em NDC
(Normalized Device Coordinate)



Projeção Perspectiva

$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ -1 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix}$$

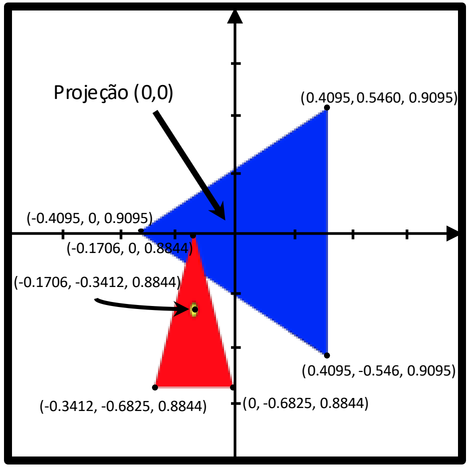
Projeção Perspectiva

$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} -2 & 0 & -1 \\ -2 & -2 & 0 \\ -8 & -8 & -8 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix}$$

Projeção Perspectiva

$$\begin{bmatrix} 1.3649 & 0 & 0 & 0 \\ 0 & 2.7298 & 0 & 0 \\ 0 & 0 & -1.01 & -1.005 \\ 0 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 3 & -3 \\ -2 & 2 & 0 \\ -10 & -10 & -10 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix}$$

Aplicando Divisão Homogênea



Projeção em NDC
(Normalized Device Coordinate)

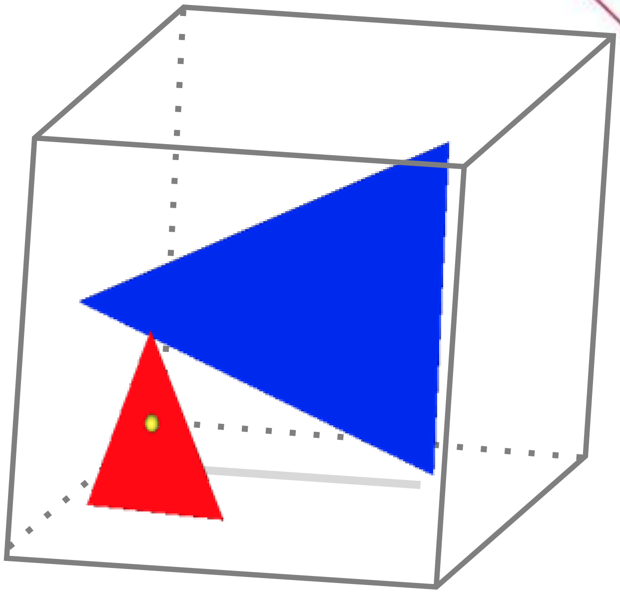
$$\begin{bmatrix} -1.3649 \\ -2.7298 \\ 7.075 \\ 8 \end{bmatrix} \mapsto \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2.7298 & 0 & -1.3649 \\ -5.4596 & -5.4596 & 0 \\ 7.075 & 7.075 & 7.075 \\ 8 & 8 & 8 \end{bmatrix} \mapsto \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4.0947 & 4.0947 & -4.0947 \\ -5.4596 & 5.4596 & 0 \\ 9.095 & 9.095 & 9.095 \\ 10 & 10 & 10 \end{bmatrix} \mapsto \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix}$$

Aplicando Divisão Homogênea

Projeção em NDC
(Normalized Device Coordinate)



$$\begin{bmatrix}-1.3649\\-2.7298\\7.075\\8\end{bmatrix} \mapsto \begin{bmatrix}-0.1706\\-0.3412\\0.8844\\1\end{bmatrix}$$

$$\begin{bmatrix}-2.7298 & 0 & -1.3649\\-5.4596 & -5.4596 & 0\\7.075 & 7.075 & 7.075\\8 & 8 & 8\end{bmatrix} \mapsto \begin{bmatrix}-0.3412 & 0 & -0.1706\\-0.6825 & -0.6825 & 0\\0.8844 & 0.8844 & 0.8844\\1 & 1 & 1\end{bmatrix}$$

$$\begin{bmatrix}-4.0947 & 4.0947 & -4.0947\\-5.4596 & 5.4596 & 0\\9.095 & 9.095 & 9.095\\10 & 10 & 10\end{bmatrix} \mapsto \begin{bmatrix}-0.4095 & 0.4095 & -0.4095\\-0.5460 & 0.5460 & 0\\0.9095 & 0.9095 & 0.9095\\1 & 1 & 1\end{bmatrix}$$

Mapeando coordenadas para tela (screen)

Escala para (W/2, H/2) Translate por (1,1) Espelha em Y

$$\begin{bmatrix} \frac{W}{2} & 0 & 0 & 1 \\ 0 & \frac{H}{2} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{W}{2} & 0 & 0 & \frac{W}{2} \\ 0 & -\frac{H}{2} & 0 & \frac{H}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Resolução = 40 x 20

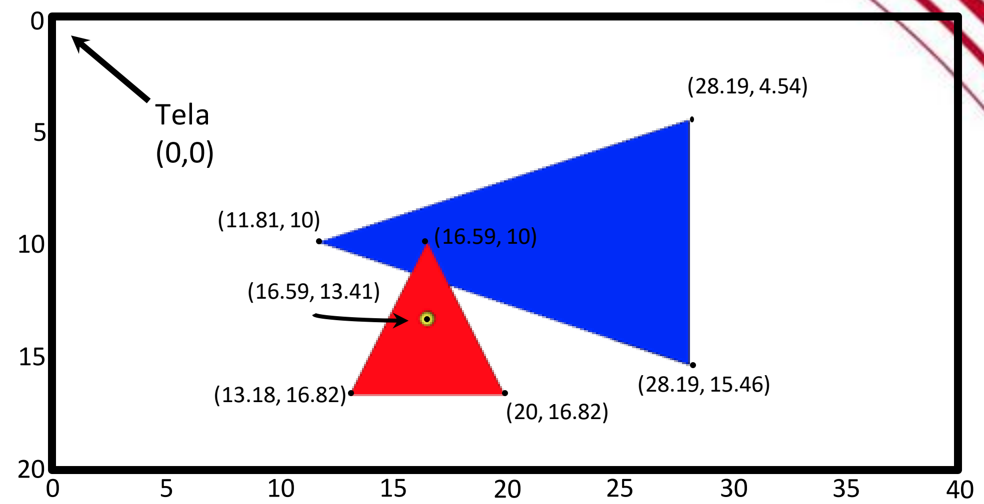
$S_{40,20}$ $T_{1,1}$ E_y

$$\text{Coordenadas da Tela} = \begin{bmatrix} \frac{40}{2} & 0 & 0 & 0 \\ 0 & \frac{20}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{20} & \mathbf{0} & \mathbf{0} & \mathbf{20} \\ \mathbf{0} & \mathbf{-10} & \mathbf{0} & \mathbf{10} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Obs: Não há necessidade da matriz ser 4x4.

Coordenadas de Tela

```xml
<Scene>
  <Viewpoint position="-8 1 1" orientation="0 1 0 -1.57"/>
  <Transform>
    <Shape>
      <Sphere radius='0.1'/>
      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```



Tela

$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.1706 \\ -0.3412 \\ 0.8844 \\ 1 \end{bmatrix} = \begin{bmatrix} 16.59 \\ 13.41 \\ 0.88 \\ 1 \end{bmatrix}$$

Tela

$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.3412 & 0 & -0.1706 \\ -0.6825 & -0.6825 & 0 \\ 0.8844 & 0.8844 & 0.8844 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 13.18 & 20 & 16.59 \\ 16.82 & 16.82 & 10 \\ 0.88 & 0.88 & 0.88 \\ 1 & 1 & 1 \end{bmatrix}$$

Tela

$$\begin{bmatrix} 20 & 0 & 0 & 20 \\ 0 & -10 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -0.4095 & 0.4095 & -0.4095 \\ -0.5460 & 0.5460 & 0 \\ 0.9095 & 0.9095 & 0.9095 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 28.19 & 28.19 & 11.81 \\ 15.46 & 4.54 & 10 \\ 0.91 & 0.91 & 0.91 \\ 1 & 1 & 1 \end{bmatrix}$$

Renderização

```
<Scene>
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  <Transform>
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      <Appearance>
        <Material diffuseColor='1 1 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform>
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='1 0 0'/>
      </Appearance>
    </Shape>
  </Transform>
  <Transform scale="1 3 2"
          rotation="1 0 0 -1.57"
          translation="2 1 1">
    <Shape>
      <TriangleSet>
        <Coordinate point='0 -1 -1 0 -1 1 0 1 0'/>
      </TriangleSet>
      <Appearance>
        <Material diffuseColor='0 0 1'/>
      </Appearance>
    </Shape>
  </Transform>
</Scene>
```
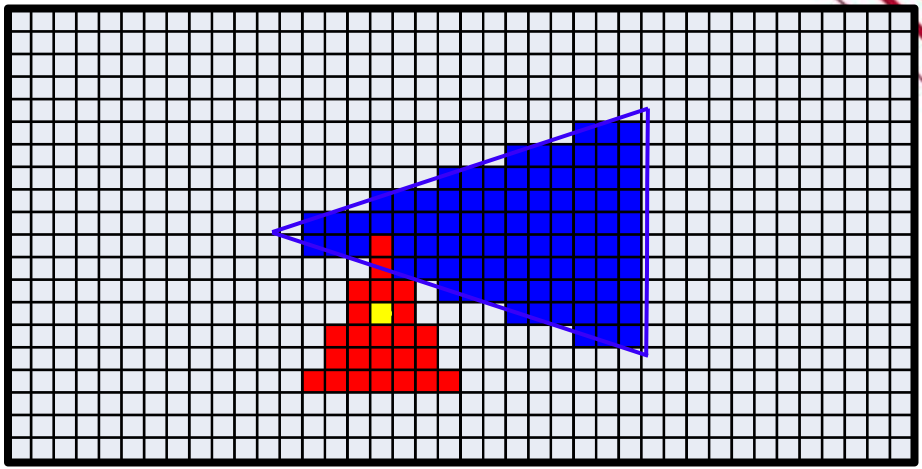



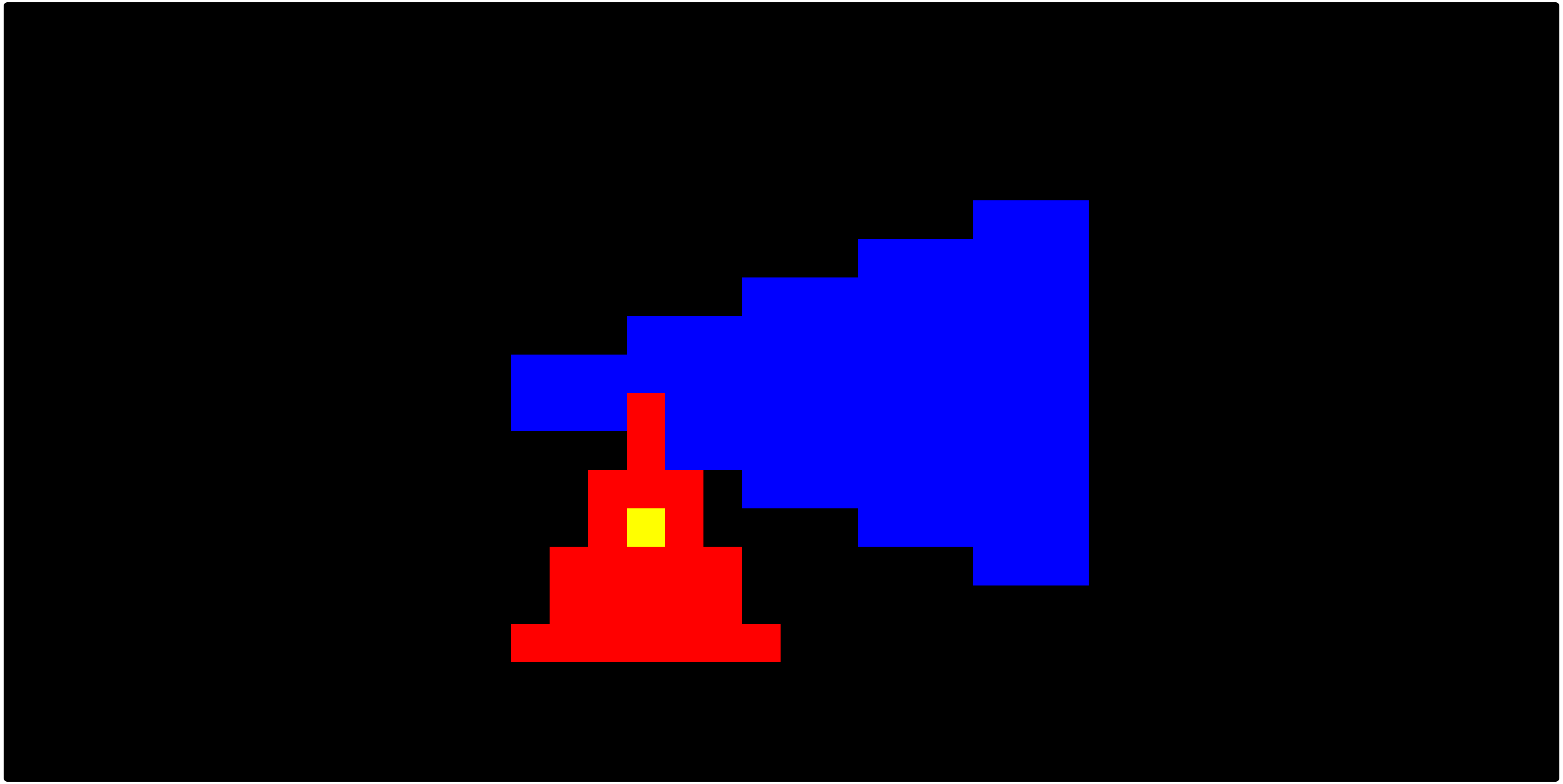
Rasterizar os Triângulos

$$Esfera: \begin{bmatrix} 16.59 \\ 13.41 \end{bmatrix}$$

$$Triângulo\ Vermelho: \begin{bmatrix} 13.18 & 20 & 16.59 \\ 16.82 & 16.82 & 10 \end{bmatrix}$$

$$Triângulo\ Azul: \begin{bmatrix} 28.19 & 28.19 & 11.81 \\ 15.46 & 4.54 & 10 \end{bmatrix}$$

Resultado Final (sem anti-aliasing)



Resolução Final: 40 x 20 pixels

Revisão Numpy

- **numpy.array**: cria matrizes tipo numpy
- **numpy.matmul**: multiplica matrizes numpy
  - A partir do Python 3.5 você pode usar o arroba (**@**)
- ***** : multiplica todos os valores da matriz por um escalar
  - **numpy.multiply** se quiser fazer uma chamada de função
- **/** : divide todos os valores da matriz por um escalar
- **numpy.empty**: matriz iniciada com valores não iniciados
- **numpy.zeros**: matriz com todos os valores sendo zero
- **numpy.ones**: matriz com todos os valores sendo um
- **np.uint8, np.uint16, np.float32**: tipos de dados para numpy

CUIDADO

- **numpy.dot**: realiza o cálculo do produto escalar
  (mas também algumas multiplicações de matrizes)
  (mas é confuso e não funciona em todos os casos)

Computação Gráfica

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